The method of discretizations in existence problems for stochastic evolution equations

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Abstract

Combining the tightness criteria of S-topology [2] and the limit theorems for stochastic integrals [1] leads to a nice proof of existence of solutions of both stochastic differential equations and the stochastic evolution equations under quite moderate assumptions.

The situation is more complicated in the case of evolution equations in separable Hilbert spaces with multiplicative noise given by a general cylindrical Lévy process. Basically, at this level of generality only the stochastic integral [3] is given and all other tools must be developed.

In this *work in progress* (joint with M. Riedle) we search for the existence of solutions to the equation

$$X(t) = S(t)x_0 + \int_0^t S(t-s)F(X(s))\,ds + \int_0^t S(t-s)G(X(s-))\,dL(s),$$

where $\{S(t)\}$ is a strongly continuous semigroup on a separable Hilbert space V, L is a cylindrical Lévy process on a separable Hilbert space U, $F : V \to V$ is a continuous (nonlinear) mapping of V and $G : V \to \mathcal{L}_2(U, V)$ is a continuous nonlinear mapping with values in the space of Hilbert-Schmidt operators from U to V.

References

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[3] A. Jakubowski and M. Riedle, Stochastic integration with respect to cylindrical Lévy processes, *The Annals of Probability*, **Vol. 45 (2017)**, 4273–4306.