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Construction of Mac-Kean and Boltzmann type flows: the sewing lemma approach

Abstract : We are concerned with Boltzmann type equations, this means (in probabilistic terms) equations driven by a Poisson point measures with the intensity depending on the law of the solution itself. Both the analytical Boltzmann equation and the probabilistic interpretation initiated by Tanaka have intensively been discussed in the literature for specific models related to the behavior of gas molecules. In this paper we first consider general abstract coefficients and then we discuss the link with specific models as well. In contrast with the usual approach in which integral equations are used in order to state the problem, we employ here a new formulation of the problem in terms of flows of endomorphisms on the space of probability measure endowed with the Wasserstein distance. This point of view already appeared in the framework of rough differential equations. Our results concern existence and uniqueness of the solution, in the formulation of flows, but we also prove that the "flow solution" is a solution of the classical integral weak equation and we obtain an uniqueness result (both for the homogenous and in homogeneous classical Bolzmann equation) which is a slight generalization of the already known results. Moreover we obtain stability results and regularity with respect to the time for such solutions. Finally we prove the convergence of empirical measures based on particle systems to the solution of our problem, and we obtain the rate of convergence. We discuss as examples the homogeneous and the in-homogeneous Boltzmann (Enskog) equation with hard potentials. Last but not list, we are able to insert a Mac Kean-Vlasov type effect: the coefficients of our equation are allowed to depend on the law of the solution as well.