Stéphane Menozzi

Université d'Evry

Convergence rate of the Euler-Maruyama scheme applied to diffusion processes with $L^q - L^\rho$ drift coefficient and additive noise

Abstract : We are interested in the time discretization of stochastic differential equations with additive d-dimensional Brownian noise and $L^q - L^\rho$ drift coefficient when the condition $\frac{d}{\rho} + \frac{2}{q} < 1$, under which Krylov and Röckner (PTRF, 2005) proved existence of a unique strong solution, is met. We show weak convergence with order $\frac{1}{2}(1 - (\frac{d}{\rho} + \frac{2}{q}))$ which corresponds to half the distance to the threshold for the Euler scheme with randomized time variable and cutoffed drift coefficient so that its contribution on each time-step does not dominate the Brownian contribution. More precisely, we prove that both the diffusion and this Euler scheme admit transition densities and that the difference between these densities is bounded from above by the time-step to this order multiplied by some centered Gaussian density.